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Coupled-channel analysis of isoscalar f_0-mesons in $\bar{N}N$ annihilation and $\pi\pi$ scattering

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Abstract. The properties of $f_0(980)$, $f_0(1370)$ and $f_0(1500)$ mesons are studied by means of a generalization of the Breit-Wigner approach preserving unitarity in case of several overlapping resonances.

PACS. 11.80.-m Relativistic scattering theory – 13.25.Jx Decays of other mesons – 13.75.Lb Meson-meson interactions – 14.40.Cs Other mesons with $S = C = 0$, mass < 2.5 GeV

1 Introduction

A goal of this work is to find the parameters of f_0-mesons using a method appropriate for broad overlapping resonances. The obvious advantage of the Breit-Wigner (BW) approach is that its parameters have a direct physical meaning. Simple BW parameterization is applicable only in case of well-separated resonances, far away from the channels' thresholds. In this generalization, the multichannel scattering amplitudes are sums of BW terms with relative phases that are not free parameters but determined by the unitarity relations. The partial and total widths may be energy dependent; they are approximately constant only if the resonances are far away from all thresholds.

We also show how to apply this BW method for production processes and use it to describe the f_0 states in $\bar{N}N$ annihilation.

2 Unitary BW S-matrix for overlapping resonances

In a previous work [1], we developed a generalization of the BW method applied to the problem of several resonances which could be strongly overlapping. This work also includes a comparison of different methods suitable for application to overlapping resonances. An approach has been to write down the S-matrix as a sum of resonance terms, and then to try to solve the constraints imposed by unitarity on the resonance parameters. The K-matrix method in which unitarity is satisfied is a common way used to describe experimental data. However, the "physical" states and the poles in the K-matrix formalism are not equivalent, the "physical" particles are more closely related to BW description. Thus, writing down the S-matrix as a sum of resonance terms with no redundant parameters could give a more direct way to extract the resonances parameters from data. Our purpose is to give an algebraic derivation of the unitarity constraints and their solutions for the BW approach. Having done this, the formalism can be applied to production reactions in which the overlapping resonances appear in the final states, together with another particle, or particles (in a similar manner to the K-matrix method [2]). Some features of our method are close to the approach of work [3].

For scalar mesons the threshold energy dependence is very important. In the development of work [1] the scattering matrix contains phase space factors, $\rho_k(E)$:

$$S(E) = I - i\sqrt{\rho(E)}\,T(E)\,\sqrt{\rho(E)}. \tag{1}$$

Lets write $T(E)$ in a resonance form

$$T = \sum_{r=1}^{N} \frac{\boldsymbol{g}_r \boldsymbol{g}_r}{E - \varepsilon_r(E)}, \qquad T_{ij} = \sum_{r=1}^{N} \mathrm{e}^{i\varphi_{ij}^{(r)}} \frac{|g_{ir}| \cdot |g_{rj}|}{E - \varepsilon_r(E)}\,,$$

where $\boldsymbol{g}_r \equiv \boldsymbol{g}_r^x + i\boldsymbol{g}_r^y$ are complex, energy-independent N-components vectors (N is the number of coupled channels), g_{ri} are coupling constants, $\varepsilon_r(E) \equiv \varepsilon_r^x(E) + i\varepsilon_r^y(E)$. The interference of the resonances with the same decay channels is the key aspect of any analysis and interpretation. In the BW approach this interference is often taken into account by relative phases in BW terms which are treated as free parameters, the most often just 0 or π. The results of analysis critically depend on the choice of this phase set. Whether or not these phases are included, such a sum of BW terms is depleted of unitarity which is the basic point in the BW description. In our approach, real energy-independent phases φ_{ri} are not free parameters but

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should be determined in such a way which preserves unitarity. The scattering matrix T is free of threshold singularities which are included in the diagonal $M \times M$ matrix $\rho(E)$ with the elements $\rho_k(E)$. Function $\rho_k(E)$ is imaginary below the k-th threshold, and real when $E \geq E_k$. (We use energy E rather than $s = E^2$ only in order to simplify some formulae.) A general way to include background is described in work [1] and in the case when it is elastic, formula (1) for S_{ij} should be multiplied by $e^{i(\delta_i^B + \delta_j^B)}$ (background phases can depend on energy).

To find the relations that should be imposed on the vectors $\boldsymbol{g}_r$ to keep the matrix S unitary and symmetric, we compose their imaginary parts through the real ones:

$$\boldsymbol{g}_r^y = u_{r1}\boldsymbol{g}_1^x + u_{r2}\boldsymbol{g}_2^x + ... + u_{rN}\boldsymbol{g}_N^x, \quad (r = 1, ..., N),$$

where $U = \{u_{rk}\}_{r,k=1}^N$ is a real antisymmetric matrix. Vectors $\boldsymbol{g}_r$ satisfy the relations

$$\sum_{k=1}^M \rho_k(E)\theta_k(E)\,|g_{rk}|^2 = -\frac{2}{S}[S + 2Q_r]\varepsilon_r^y,$$

$$\sum_{k=1}^M \rho_k\theta_k \mathrm{Re}(g_{qk}^* g_{rk}) = -\frac{2}{S}[F_{qr}(\varepsilon_q^x - \varepsilon_r^x) + G_{qr}(\varepsilon_q^y + \varepsilon_r^y)],$$

$$\sum_{k=1}^M \rho_k\theta_k \mathrm{Im}(g_{qk}^* g_{rk}) = -\frac{2}{S}[G_{qr}(\varepsilon_q^x - \varepsilon_r^x) - F_{qr}(\varepsilon_q^y + \varepsilon_r^y)].$$

Here $r = 1, ..., N$, $q = r+1, ..., N$ and $\theta_k(E)$ is the step function: $\theta_k(E) = 0$, if $E < E_k$; $\theta_k(E) = 1$, if $E \geq E_k$. The (constant) coefficients S, Q_r, F_{qr} G_{qr} are determined via the elements of the coupling matrix U (see [1]).

Matrix U gives a measure of resonances overlap. If $|m_r - m_{r'}| \gg \Gamma_r + \Gamma_{r'}$, the matrix elements $u_{rk} \to 0$ and vectors $\boldsymbol{g}_r$ are getting real and orthogonal: $g_r = g_r^x$, $(\boldsymbol{g}_r\boldsymbol{g}_q) = 0$, thus we obtain an expression similar to Flatte's one-resonance formula [4]:

$$T_{ij} = \sum_{r=1}^N \frac{g_{ir}g_{rj}}{E - m_r + \frac{i}{2}\sum\limits_{k=1}^M \rho_k(g_{rk})^2}\,. \tag{2}$$

In the general case unitarity provides the expressions for energy-dependent masses and widths:

$$\Gamma_r(E) = -2\varepsilon_r^y(E) = \frac{S}{S + 2Q_r}\sum_{k=1}^M \rho_k(E)\theta_k(E)\,|g_{rk}|^2,$$

$$\varepsilon_r^x(E) = \varepsilon_1^x(E) + \frac{S}{2(F_{1r}^2 + G_{1r}^2)}\sum_{k=1}^M \rho_k(E)\theta_k(E) \times [F_{1r}\mathrm{Re}(g_{1k}^* g_{rk}) + G_{1r}\mathrm{Im}(g_{1k}^* g_{rk})]\,,$$

$$\varepsilon_1^x(E) = m_1 - i\,\frac{S}{2(S + 2Q_1)}\sum_{k=1}^M \rho_k(E)(1 - \theta_k(E))|g_{1k}|^2.$$

m_1 is the "bare" mass in the first BW term. For three resonances the coefficients in these formulae are:

$$S = 1 - \alpha^2 - \beta^2 - \gamma^2,$$
$$Q_1 = \alpha^2 + \beta^2, \quad Q_2 = \alpha^2 + \gamma^2, \quad Q_3 = \beta^2 + \gamma^2,$$
$$F_{12} = -\alpha, \quad F_{13} = -\beta, \quad F_{23} = -\gamma,$$
$$G_{12} = \beta\gamma, \quad G_{13} = -\alpha\gamma, \quad G_{23} = \alpha\beta.$$

Real parameters α, β, γ are the elements of the matrix

$$U = \begin{pmatrix} 0 & -\alpha & -\beta \\ \alpha & 0 & -\gamma \\ \beta & \gamma & 0 \end{pmatrix},$$

and are restricted by the relation $\alpha^2 + \beta^2 + \gamma^2 < 1$. The rest of the constraints on $\boldsymbol{g}_r$ are quadratic equations for each $k = 1, ..., M$.

It is simple to consider the slightly more general case in which g_{ir} is replaced by f_{pr}, where p (the production channel, $\bar{N}N$ in our case) does not occur in the sums in $\Gamma_r(E)$ and $\varepsilon_r(E)$. We would then have for the transition from the production state p to final state k

$$F_{pk} = \sum_{r=1}^N \frac{f_{pr}g_{rk}}{E - \varepsilon_r^x(E) + \frac{i}{2}\Gamma_r(E)}\,. \tag{3}$$

Vectors $\boldsymbol{f}_r$ like vectors $\boldsymbol{g}_r$ are complex. The amplitudes F have the same poles as the amplitudes T.

The "running mass" $\varepsilon_r^x(E)$ is approximately constant only in case when all the resonances are far away from thresholds. The equation $E - \varepsilon_r^x(E) = 0$ gives the mass of the r-th resonance, m_r (generally it can have several roots, but for f_0 states we obtain one root for each r).

Branching ratios of the decay of the r-th resonance into the k-th channel are given by

$$B_{rk} = \frac{\Gamma_{rk}}{\Gamma_r} = \frac{\rho_k(m_r)\theta_k(m_r)\,|g_{rk}|^2}{\sum\limits_{k=1}^M \rho_k(m_r)\theta_k(m_r)\,|g_{rk}|^2}\,. \tag{4}$$

3 Analysis of $f_0(980)$, $f_0(1370)$, $f_0(1500)$ states

Similar to a number of other analyses, our model includes four channels: $\pi\pi$, $\bar{K}K$, $\eta\eta$, 4π. We do not include the $\eta\eta'$-channel due to lack of data. Phase state factors are:

$$\rho_k(s) = \sqrt{(s - s_k)/s},$$
$$s_1 = 4m_\pi^2, \quad s_2 = 4m_K^2, \quad s_3 = 4m_\eta^2,$$
$$\rho_4(s) = \sqrt{(s - s_4)/s}/\left(1 + \exp\left[\Lambda \cdot (s_0 - s)\right]\right),$$
$$s_4 = 16m_\pi^2, \quad s_0 = 2.8\,(\mathrm{GeV}^{-2}), \quad \Lambda = 3.5\,(\mathrm{GeV}^{-2}).$$

For ρ_4 we use the strongly energy-dependent 4π phase factor [5] that approximates either the $\rho\rho$ or $\sigma\sigma$ phase space. With a small change in the parameter s_0 one can assume that the 4π simulates all inelastic channels not included in our scheme directly.

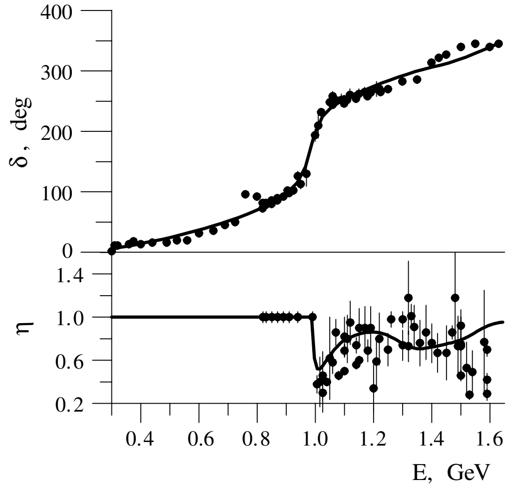

Fig. 1. Phase δ and inelasticity η for $I = J = 0$ $\pi\pi$ scattering. Experimental points are from [6].

Table 1. Branching ratios (in %) and relative to $f_{0.98}$ phases (in degrees).

State	$B_{f_{0.98}}$	$B_{f_{1.37}}$	$\varphi_{f_{1.37}}$	$B_{f_{1.50}}$	$\varphi_{f_{1.50}}$
$\pi\pi$	77.3	8.2	−28.9	4.0	28.4
$\bar{K}K$	20.8	5.8	−38.0	3.4	19.2
$\eta\eta$	1.9	52.7	68.2	57.8	16.6
4π	0.01	33.3	88.7	34.8	26.5

Figure 1 shows the $\pi\pi$ S-wave within the model including three f_0 states. Ignoring complications connected with identical pions in the final states, the formula which was used in a number of paper (see [7]) for the resonant mass spectra in the reactions $\bar{N}N \to (k)\pi^0$ can be written as

$$\frac{\mathrm{d}N_k}{\mathrm{d}m} = C_k \sqrt{\left[1 - \frac{(m - m_\pi)^2}{4m_N^2}\right]\left[1 - \frac{(m + m_\pi)^2}{4m_N^2}\right]} \frac{2m}{\pi} \times |\rho_k(m) F_{pk}(m)|^2,$$

where F_{pk} are the production amplitudes (3). The fit of some Crystal Barrel data is shown in figs. 2 and 3.

Masses and widths of f_0 states (in GeV) are

$$m_1 = 0.987, \quad m_2 = 1.356, \quad m_3 = 1.549,$$
$$\Gamma_1 = 0.117, \quad \Gamma_2 = 0.269, \quad \Gamma_3 = 0.151.$$

These numbers are compatible with previous findings [8]. Table 1 contains other parameters having physical meaning. The number of free parameters in the unitarized BW method is MN comparing to $N(M+1)$ in a naive BW (if we take a sum of terms (2)) and K-matrix methods. Here we use the "bare" mass m_1, coefficients α, β, γ, and 8 out of 12 real parts of vectors g_{ir} as free parameters. The fit of $\bar{N}N$ annihilation processes also requires N complex numbers f_{pr}. The $\pi\pi$ background phase shift is a linear function of energy and is about 90° near $\bar{K}K$ threshold.

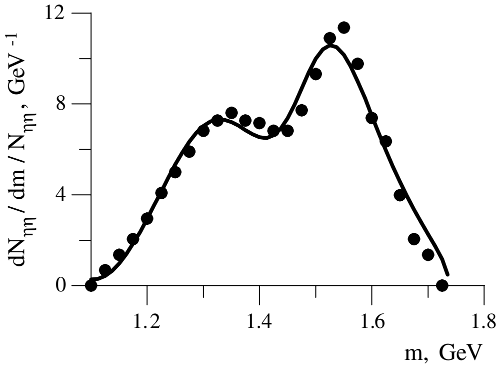

Fig. 2. Normalized $\eta\eta$ mass spectra in $\bar{p}p \to (\eta\eta)\pi^0$ at rest. The data are from [9].

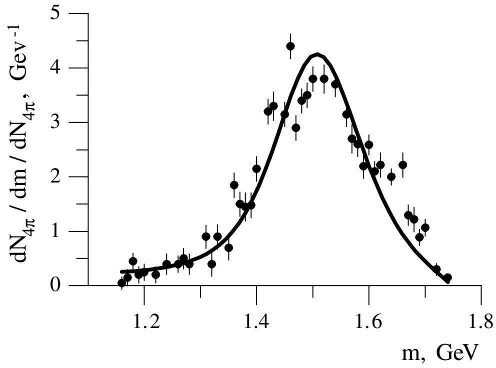

Fig. 3. Normalized $4\pi^0$ mass spectra in $\bar{N}N \to (4\pi^0)\pi^0$ at rest. The data are from [10].

4 Conclusion

We present a scheme of unitarization of a sum of BW terms which can be useful to the study of several overlapping resonances within the same partial wave. The resulting formula (1) resembles a traditional sum of BW terms with the relative phases that are not free parameters but are determined from the unitarity constraints.

We apply this approach to the analysis of the scalar f_0-mesons in the $\pi\pi$ scattering and $\bar{N}N$ annihilation at rest. The results show that $f_0(1370)$ and $f_0(1500)$ states strongly interfere. This stresses the necessity to use unitary description. Masses and widths of these states are compatible with previous findings. In a model with four coupled channels, the main decay modes for $f_0(1370)$ and $f_0(1500)$ are $\eta\eta$ and 4π, while the $\pi\pi$ and $\bar{K}K$ decays are suppressed.

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